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The influence of low-conductivity boundaries on time-dependent convection

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The influence of the thermal conductivity of rigid boundaries is analysed for the case when the onset of convection occurs in the form of a time-dependent mode. A new form of long-wavelength convection is found which is disjunct from the high-wavenumber thermal Rossby wave in the presence of infinitely conducting boundaries usually considered.

1. Introduction

It is well known that the ratio λ between the thermal conductivity of the boundary and that of the fluid has a profound influence on the onset of convection (Sparrow, Goldstein & Jonsson 1964) and on its nonlinear evolution (Busse & Riahi 1980; Proctor 1981; Riahi 1985). In all cases that have been investigated hitherto, however, attention has been focused on steady convection flows. In the case of time-dependent convection flows new aspects enter the problem and, in particular, the phase lag of the fluctuating component of the temperature field in the boundary must be taken into account. This subject is addressed in this paper where the case of thermal Rossby waves in a rotating cylindrical annulus is considered as an example.

Convection in the fluid gap between two rotating coaxial cylinders, the inner (outer) of which is cooled (heated), is driven by centrifugal buoyancy if the angular velocity Ω is high enough such that $\Omega^2 r_0 \gg g$ where r_0 is the mean radius of the annular region and g is the acceleration due to gravity. When a configuration is used as shown in figure 1, the height of the fluid region varies with distance from the axis and the onset of convection in the form of steady motions is no longer possible. Instead the convection rolls or columns aligned with the axis of rotation exhibit the dynamical properties of Rossby waves and propagate in the prograde direction. They thus provide a convenient example for the study of the influence of boundaries of low thermal conductivity in the case of time-dependent convection.

Convection in the form of thermal Rossby waves has been studied experimentally (Busse & Carrigan 1974; Busse *et al.* 1997). Usually the conductivity of the boundaries is much higher than that of the fluid. But recently laboratory observations of thermal Rossby waves in liquid metals in an apparatus such as sketched in figure 1 have been carried out by Jaletzky (1999). Although the analysis of this paper focuses on the mathematical structure of the problem and is not suited for a qualitative comparison with the measured data, the experiment has provided an additional motivation for the analysis.

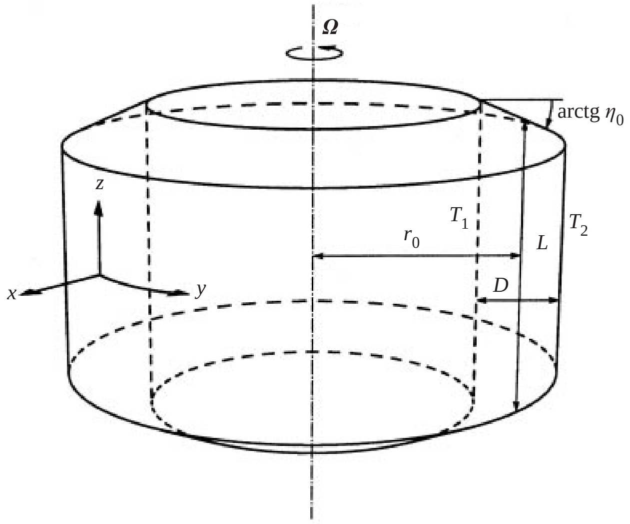

FIGURE 1. Sketch of the geometrical configuration of the rotating cylindrical annulus.

2. Mathematical formulation of the problem

We consider the rotating cylindrical annulus as shown in figure 1. Using the thickness D of the annular gap as length scale, D^2/ν as time scale where ν is the kinematic viscosity, and $P(T_2 - T_1)/R$ as temperature scale where T_1 and T_2 are the mean temperatures of the inner and outer boundaries, we can write the equations for the stream function ψ and the deviation Θ of the temperature from its static distribution in the form

$$\left(\frac{\partial}{\partial t} - \Delta_2\right)\Delta_2\psi - \eta\frac{\partial}{\partial y}\psi + \frac{\partial}{\partial y}\Theta = \left(\frac{\partial}{\partial x}\psi\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\psi\frac{\partial}{\partial x}\right)\Delta_2\psi, \tag{2.1a}$$

$$\left(P\frac{\partial}{\partial t} - \Delta_2\right)\Theta + R\frac{\partial}{\partial y}\psi = P\left(\frac{\partial}{\partial x}\psi\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\psi\frac{\partial}{\partial x}\right)\Theta, \tag{2.1b}$$

where the Rayleigh number R, the Prandtl number P and the Coriolis parameter η are defined by

$$R = \frac{\gamma(T_2 - T_1)\Omega^2 r_0 D^3}{\nu\kappa}, \quad P = \frac{\nu}{\kappa}, \quad \eta = \frac{4\eta_0\Omega D^3}{\nu L}.$$

Here γ and κ denote the thermal expansivity and thermal diffusivity of the fluid, L is the average height of the fluid annulus and arctg η_0 is the angle of the conical boundary as indicated in figure 1. Assuming the narrow gap limit of the annulus, we have introduced a Cartesian coordinate system with x, y and z in the radial, azimuthal and axial directions, respectively. Δ_2 stands for the Laplacian in the (x, y)-plane. In the case of plane boundaries, arctg $\eta_0 = 0$, the velocity field $\boldsymbol{v}$ can be described entirely by the stream functions $\psi(x, y, t)$ with $v_x = A\partial_y\psi, v_y = -A\partial_x\psi$ where A denotes an amplitude factor which we do not need to specify at this point. In the limit arctg $\eta_0 \ll 1$ the z-component of the velocity enters through the condition of vanishing normal velocity at the conical boundaries and gives rise to the term with η in equation (2.1a) after the equation for the vertical component of vorticity, $\Delta_2\psi$, is averaged over the z-coordinate. Since $\Omega D^2/\nu \gg 1$ is assumed a small value of arctg

η_0 is in accordance with a finite or even large value of η. For further details on the derivation of equations (2.1) and their range of validity we refer to Busse (1986).

At the cylindrical walls no-slip conditions and a finite thermal conductivity will be assumed,

$$\psi = \frac{\partial}{\partial x}\psi = \Theta - \Theta^e = \frac{\partial}{\partial x}\Theta - \lambda\frac{\partial}{\partial x}\Theta^e = 0 \quad \text{at} \quad x = \pm\tfrac{1}{2}, \tag{2.2}$$

where Θ^e denotes the deviation of the temperature from the static distribution in the boundaries. Unless stated otherwise we shall restrict attention to the linear problem of the onset of convection, for which the right-hand sides of equations (2.1) can be dropped. Without losing generality we assume the y- and t-dependence of the solution in the form $\exp\{\mathrm{i}\alpha y - \mathrm{i}\omega t\}$. There is no need to denote this dependence explicitly in the linear analysis of the problem if the derivatives $\partial/\partial t$ and $\partial/\partial y$ in equations (2.1) are just replaced by $-\mathrm{i}\omega$ and $\mathrm{i}\alpha$, respectively. The temperature distribution Θ^e within the walls can then be readily derived,

$$\Theta^e = \Theta_0^e \exp\{\mp(\alpha^2 - \mathrm{i}\omega P\beta)^{1/2}(x \mp \tfrac{1}{2})\} \quad \text{for} \quad x \gtrless \pm\tfrac{1}{2}, \tag{2.3}$$

where infinitely thick walls have been assumed and β denotes κ divided by the thermal diffusivity of the boundary. Using solution (2.3) we can eliminate Θ^e from the boundary conditions (2.2) and wrie

$$\partial_x\Theta = \mp\lambda(\alpha^2 - \mathrm{i}\omega P\beta)^{1/2}\Theta \quad \text{at} \quad x = \pm\tfrac{1}{2}. \tag{2.4}$$

If the walls are not infinitely thick, but exceed the gap width d by the factor δ, then a boundary condition of the form (2.4) can still be derived with λ replaced by $\lambda^* = \lambda \coth(\alpha^2 - \beta\mathrm{i}\omega P)^{1/2}\delta$. As long as the argument of coth is not too small, this replacement will not change the analysis described below in a substantial way. In a similar way effects of finite curvature of the annulus could eventually be taken into account.

3. Analysis of the linear problem

In order to obtain an asymptotic analytical solution of the linearized version of equations (2.1) together with boundary conditions (2.2) and (2.4) we introduce an expansion

$$\Theta = \Theta_0 + \Theta_1 + \Theta_2 + \ldots, \qquad \psi = \psi_1 + \psi_2 + \ldots, \tag{3.1}$$

where each succeeding term is asymptotically small in comparison with the preceding one. We start by assuming that λ and α are small quantities in order to obtain

$$\Theta_0 = 1$$

just as one would get in the case arctg $\eta_0 = 0$ with $\omega = 0$. As a result we obtain the equation

$$\left(\frac{\partial^4}{\partial x^4} + \mathrm{i}\alpha\eta\right)\psi_1 = \mathrm{i}\alpha \tag{3.2a}$$

for ψ_1 which together with condition (2.2) is solved by

$$\psi_1 = \eta^{-1}\left[1 - \frac{\mu_2 \sinh\frac{1}{2}\mu_2 \cosh\mu_1 x - \mu_1 \sinh\frac{1}{2}\mu_1 \cosh\mu_2 x}{\mu_2 \sinh\frac{1}{2}\mu_2 \cosh\frac{1}{2}\mu_1 - \mu_1 \sinh\frac{1}{2}\mu_1 \cosh\frac{1}{2}\mu_2}\right], \tag{3.2b}$$

where μ_1 and μ_2 are the roots of $\mu^4 = -\mathrm{i}\alpha\eta$ with positive real parts,

$$\mu_1 = \exp\{\mathrm{i}3\pi/8\}(\alpha\eta)^{1/4}, \qquad \mu_2 = \exp\{-\mathrm{i}\pi/8\}(\alpha\eta)^{1/4}. \tag{3.2c}$$

For Θ_1 we now obtain the equation

$$\frac{\partial^2}{\partial x^2}\Theta_1 = \alpha^2 - \mathrm{i}\omega P + \mathrm{i}\alpha R \psi_1 \tag{3.3a}$$

together with the boundary condition

$$\frac{\partial}{\partial x}\Theta_1 = \mp\lambda\sqrt{\alpha^2 - \mathrm{i}\omega P \beta} \quad \text{at} \quad x = \pm\tfrac{1}{2}. \tag{3.3b}$$

Multiplication of equation (3.3*a*) by Θ_0 and integration over the interval $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$ yields after two partial integrations of the left-hand side and use of condition (3.3*b*)

$$\mathrm{i}\alpha R \int_{-1/2}^{1/2} \psi_1 \mathrm{d}x + \alpha^2 = \mathrm{i}\omega P - 2\lambda\sqrt{\alpha^2 - \mathrm{i}\omega P\beta}. \tag{3.4}$$

The integral over ψ can be easily evaluated and a simple approximation can be obtained for $\alpha\eta$ of the order unity or less,

$$\int_{-1/2}^{1/2} \psi_1 \mathrm{d}x = 1 - 2\left(\frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_2}\right)\left[\mu_2 \coth\frac{\mu_1}{2} - \mu_1 \coth\frac{\mu_2}{2}\right]^{-1}$$

$$= 1 - \left(\frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_2}\right)\left[\left(\frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_2}\right)\left(1 - \frac{\mathrm{i}\alpha\eta}{720} + \frac{\alpha^2\eta^2 3}{(720)^2 7} + \cdots\right)\right], \tag{3.5a}$$

where the expansion of $\coth z$

$$\coth z = \frac{1}{z} + \frac{z}{3} - \frac{z^3}{45} + \frac{2z^5}{945} - \frac{z^7}{225\times 21} + \frac{2z^9}{243\times 385} - \cdots, \tag{3.5b}$$

has been used (Abramowitz & Stegun 1965). The solvability condition (3.4) can thus be written in the form

$$\left(\frac{R}{720} - 1\right)\alpha^2 - \mathrm{i}\alpha^3\eta\frac{R}{720\times 504} = 2\lambda\sqrt{\alpha^2 - \mathrm{i}\omega P\beta} \tag{3.6}$$

if terms of higher order than those denoted explicitly in expressions (3.5) are neglected. We have also dropped $\mathrm{i}\omega P$ from the right-hand side of equation (3.6) since this term turns out to be small compared to the others. After taking the squares of left- and right-hand sides of (3.6), evaluating real and imaginary parts separately and minimizing R with respect to α we find

$$R_c = 720\left(1 + \frac{1}{3}\left(\frac{\eta\lambda}{14}\right)^{1/2}\right), \quad \alpha_c = 12(7\lambda/\eta)^{1/2}, \quad \omega_c P = \frac{28\times 72\times\sqrt{2}\lambda}{\eta\beta}. \tag{3.7a–c}$$

A comparison with direct integrations of the linear version of equations (2.1) with the shooting method (Press *et al.* 1986) indicates that the asymptotic relations (3.7) appear to be approximately valid far beyond the expected regime. In particular they provide a good approximation for $\alpha_c\eta \approx 1$ because of the rapid convergence of expansions (3.5).

An example of the comparison of asymptotic and numerical results is shown in figure 2 where the conditions for onset of convection have been presented in the case $\eta = 10^4$ with $\beta = \lambda^{-1}$. Besides the long-wavelength convection mode described

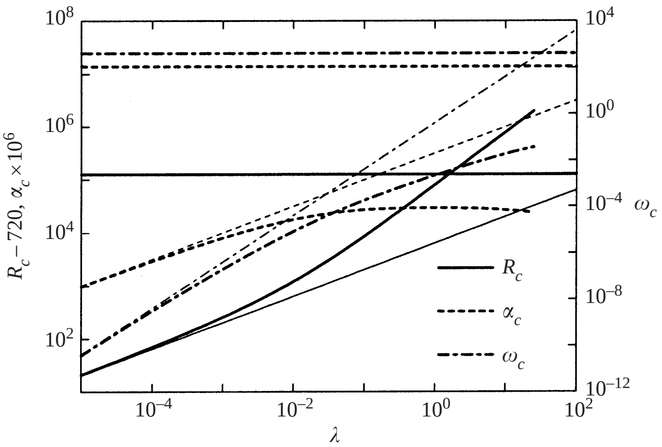

FIGURE 2. Critical values R_c, α_c and ω_c for the onset of convection as a function of the conductivity ratio λ in the case $\eta = 10^4, \beta = \lambda^{-1}$. The thin lines show the asymptotic relationships (3.7). The lines nearly parallel to the abszissa correspond to the high-wavenumber mode which is preferred for large λ.

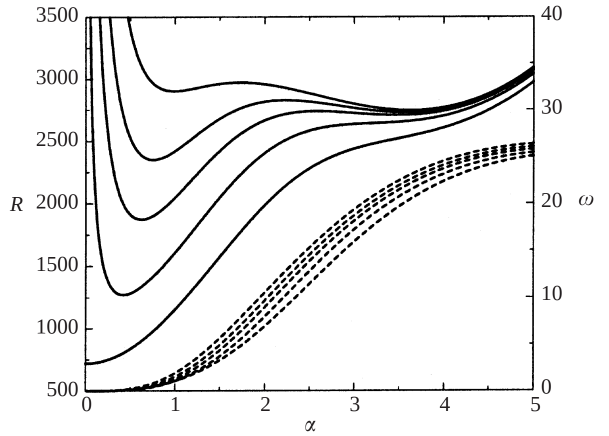

FIGURE 3. Neutral curves $R(\alpha)$ (solid, left ordinate) and $\omega(\alpha)$ (dashed, right ordinate) for $\lambda = 0.8, 0.5, 0.3, 0.1, 0$ (from top to bottom) in the case $P = 0.7, \eta = 400$.

by relationships (3.7) the high-wavenumber mode familiar from the limit of infinitely conducting boundaries (Busse 1970, 1986) is also indicated. It becomes the preferred mode for the onset of convection for $\lambda \gtrsim 1$, as expected. The appearance of two distinct modes of convection corresponding to two separate minima of the $R(\alpha)$-dependence is characteristic for sufficiently large values of η, as can be seen from figure 3 where $R(\alpha)$ has been plotted for various values of λ and η. The contrast between the two modes increases with increasing η since the wavenumber of the long-wavelength mode decreases in proportion to $\eta^{-1/2}$ while the wavenumber of the other mode is proportional to $\eta^{1/3}$. This latter property of convection with a wavelength smaller than the gap width also explains why this mode is rather insensitive to the thermal boundary conditions at the cylindrical walls as is apparent from figure 2. There is also little difference between stress-free and no-slip boundary conditions at

$x = \pm\frac{1}{2}$ for large η as has been shown by Schnaubelt & Busse (1992) and in terms of an asymptotic analysis by Plaut & Busse (2001).

The fact that the second minimum of the $R(\alpha)$-curve corresponding to the solution (3.7) disappears for low values of η is also evident from the property that the expression (3.7*b*) for the critical wavenumber diverges for $\eta \to 0$. In this limit expressions (3.7) are not valid since the neglect of $\mathrm{i}\omega P$ in equation (3.6) can no longer be justified. For $\eta \lesssim 10$ another balance becomes valid in which the term with η in the basic equations is taken into account in a higher order. Starting with the ansatz (3.1), but using tildes to distinguish the quantities from the ones previously used, we obtain instead of equation (3.2*a*)

$$\frac{\partial^4}{\partial x^4}\tilde{\psi}_1 = \mathrm{i}\alpha\tilde{\Theta}_0 = \mathrm{i}\alpha \tag{3.8a}$$

which is solved by

$$\tilde{\psi}_1 = \mathrm{i}\alpha(x^2 - \tfrac{1}{4})^2/4!. \tag{3.8b}$$

Equation (3.3*a*) for Θ_1 is replaced by

$$\frac{\partial^2}{\partial x^2}\tilde{\Theta}_1 - \alpha^2 - \mathrm{i}\alpha\tilde{R}_0\tilde{\psi}_1 = 0 \tag{3.9a}$$

which is solved by

$$\tilde{\Theta}_1 = \alpha^2\left(\frac{31}{21\times 64} - \frac{7}{16}x^2 + \frac{5}{4}x^4 - x^6\right) \tag{3.9b}$$

after the solvability condition has been satisfied with $\tilde{R}_0 = 720$ and the boundary condition $\partial\tilde{\Theta}_1/\partial x = 0$ at $x = \pm\frac{1}{2}$ has been used. The solution of the equation for $\tilde{\psi}_2$,

$$\frac{\partial^4}{\partial x^4}\tilde{\psi}_2 - 2\alpha^2\frac{\partial^2}{\partial x^2}\tilde{\psi}_1 + \mathrm{i}\alpha\eta\tilde{\psi}_1 - \mathrm{i}\alpha\tilde{\Theta}_1 = 0, \tag{3.10a}$$

is now given by

$$\tilde{\psi}_2 = \left(x^2 - \frac{1}{4}\right)^2\left[\frac{-\mathrm{i}\alpha^3}{7!}\left(x^6 - \frac{13}{4}x^4 - \frac{153}{16}x^2 + \frac{517}{64}\right) + \frac{\alpha^2\eta}{8!}\left(x^4 - \frac{11}{6}x^2 + \frac{163}{48}\right)\right] \tag{3.10b}$$

and the dependence of R on η enters through the solvability condition of the equation for $\tilde{\Theta}_2$,

$$\frac{\partial^2}{\partial x^2}\tilde{\Theta}_2 = \alpha^2\tilde{\Theta}_1 + \mathrm{i}\alpha\tilde{R}_0\tilde{\psi}_2 + \mathrm{i}\alpha\tilde{R}_1\tilde{\psi}_1 - \mathrm{i}\omega P, \tag{3.11a}$$

together with the boundary condition $\partial\tilde{\Theta}_2/\partial x = \mp\lambda\sqrt{\alpha^2 - \mathrm{i}\omega P\beta}$ at $x = \pm\frac{1}{2}$. On multiplying equation (3.11*a*) by $\tilde{\Theta}_0$ and integrating over x we obtain after using the boundary condition in evaluating the left-hand side

$$\frac{\tilde{R}_1\alpha^2}{6!} - \frac{17}{6\times 7\times 11}\alpha^4 - \frac{\mathrm{i}\eta\alpha^3}{7\times 8\times 9} + \mathrm{i}\omega P = 2\lambda\sqrt{\alpha^2 - \mathrm{i}\omega P\beta}. \tag{3.11b}$$

Anticipating $\omega P\beta \ll \alpha^2$, we can easily determine the minimum of $\tilde{R}_1$ as a function of α and the critical values for the onset of convection,

$$R_c = \min_{\alpha}(\tilde{R}_0 + \tilde{R}_1) = 720 + 2160\left(\frac{17}{6\times 7\times 11}\right)^{1/3}\lambda^{2/3}, \tag{3.12a}$$

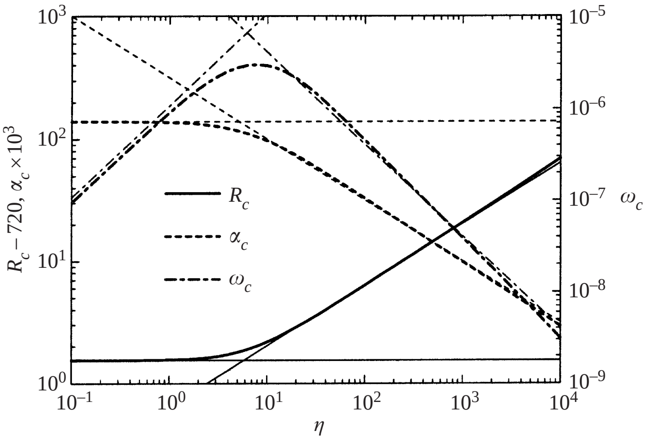

FIGURE 4. Critical values R_c, α_c and ω_c for the onset of convection as a function of the Coriolis parameter η in the case $\lambda = 10^{-4}$ with $\beta = \lambda^{-1}$. The thin lines which become asymptotic for $\eta \to 0$ correspond to expressions (3.12), while the thin lines approaching the numerical values in the intermediate regime $10 \lesssim \eta \lesssim 10^4$ correspond to expressions (3.7).

$$\alpha_c = \left(\frac{6 \times 7 \times 11}{17}\right)^{1/3} \lambda^{1/3}, \tag{3.12b}$$

$$P\omega_c = \eta\lambda \frac{11}{2 \times 17} \left(1 + \beta\lambda^{2/3} \left(\frac{17}{6 \times 7 \times 11}\right)^{1/3}\right)^{-1}. \tag{3.12c}$$

Expressions (3.12a,b) do not depend on η and are identical with those in a non-rotating Rayleigh–Bénard layer (Busse & Riahi 1980). The condition $\omega P\beta \ll \alpha^2$ is satisfied as long as $\eta \ll \min\,(\lambda^{1/3}, \lambda^{-1/3}/\beta)$. But this inequality does not have to be satisfied very strictly because large numerical factors are involved on the right-hand side.

In figure 4 numerically obtained critical values have been plotted as a function of η in comparison with the asymptotic relationships (3.7) and (3.12). The curves clearly demonstrate the changeover at about $\eta \approx 10^2\lambda^{-1/3}$ from the power laws (3.12) to the ones given by (3.7). As is also apparent from figure 4, relationships (3.7) begin to lose their validity as $\eta\lambda$ increases beyond unity since the assumption $\alpha\eta \lesssim 1$ on which expressions (3.7) are based becomes invalid. Efforts to derive still another set of asymptotic relationships based on $\alpha\eta \gg 1$ have failed and would hardly be useful since the short-wavelength mode given by

$$\alpha_c = \left(\frac{\eta P}{\sqrt{2}(1+P)}\right)^{1/3}, \quad R_c = 3\alpha_c^4, \quad \omega_c = \frac{\eta}{(1+P)\alpha_c}, \tag{3.13a–c}$$

and derived for the limit $\lambda \gg 1$ (Busse 1986; Plaut & Busse 2001) would be preferred.

4. Discussion

The new type of convection described by relationships (3.7) is unusual in that viscous dissipation appears to play a lesser role in overcoming the constraint of rotation than in the case of the thermal Rossby waves governed by (3.13) in the large-η limit. Instead the Coriolis force associated with the deviation from geostrophy is balanced primarily by thermal buoyancy. In this respect the new type of convection resembles the 'thermal mode' which has been discussed by Busse (1986) in the limit of negligible dissipation.

It is of interest to evaluate the expression (3.2*b*) for the stream function ψ_1 in the same approximation as used for (3.5),

$$\psi_1 = \frac{\mathrm{i}\alpha}{4!}\left(x^2-\frac{1}{4}\right)^2\left[1-\frac{\mathrm{i}\alpha\eta}{8!}\left(x^4-\frac{11}{6}x^2+\frac{163}{48}\right)+o\left(\alpha^2\eta^2\right)\right]. \tag{4.1}$$

As expected, ψ_1 approaches $\tilde{\psi}_1$ in the limit $\eta \to 0$. But the fact that a real part proportional to η contributes to ψ_1 has the important consequence that a mean flow proportional to the square of the amplitude of convection is generated when the nonlinear problem posed by (2.1) is considered. Since there is no mean pressure gradient directed in the azimuthal y-direction of the cylindrical annulus, the equation for the mean zonal flow $\bar{v}_y$ is given by

$$\frac{\partial^2}{\partial x^2}\bar{v}_y = \frac{\partial}{\partial x}(\overline{v_x v_y}) \approx \frac{\alpha}{2}A^2\frac{\mathrm{d}}{\mathrm{d}x}\left(\psi_r\frac{\mathrm{d}}{\mathrm{d}x}\psi_i-\psi_i\frac{\mathrm{d}}{\mathrm{d}x}\psi_r\right), \tag{4.2}$$

where the bar indicates the average over the y-coordinate and ψ_r, ψ_i denote the real and the imaginary parts of ψ. The evaluation of (4.2) yields in a first approximation

$$\bar{v}_y \approx \frac{\alpha^3\eta}{4!8!6}A^2\left(x^2-\frac{1}{4}\right)^4\left(x^4-\frac{13}{10}x^2+\frac{21}{80}\right). \tag{4.3}$$

This expression corresponds to a prograde flow with a symmetric profile with respect to the midplane of the gap. The amplitude A can be determined in terms of supercritical values of the Rayleigh number, $R - R_c$. For small values of $\alpha\eta$ the explicit expressions derived for rolls by Busse & Riahi (1980) can be used.

The result (4.3) should be seen in contrast to the mean flow of order A^4 which has been derived by Busse & Or (1986) in the case of stress-free cylindrical walls with fixed temperatures. We conclude from this discussion that not only are new modes of convection introduced by low thermal conductivities of the walls, but the dynamical properties of the convection are affected as well.

Since the goal of this paper has been the elucidation of the new dynamical balance that becomes possible in the limit of low thermal conductivity of the boundaries, we have tried to reduce the number of parameters of the problem through the consideration of limiting cases. For comparisons with experimental observations numerical determinations of critical conditions will be needed. It seems likely that the unexpected large wavelength of thermal Rossby waves seen in the experiment of Jaletzky (1999) in which λ assumes a value of the order unity can be explained quantitatively on the basis of such an analysis, which will be done in the future.

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